Statistical Analyses of Radar Reliability

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Methods showing the strictly decreasing failure rate of even old radars are considered. Mixtures of two or three Weibull distributions are used to fit complex renewal data.

Key Words

Confidence Intervals for

Weibull and Gamma Parameters

Mixtures of Weibull Distributions

1. Introduction

Decreasing failure rate results whenever the shape parameter a of the Weibull Cdf $1-\exp(-ct^a)$ is less than unity. One of the easiest-to-use methods to estimate a and obtain its confidence interval is given in [1, pp 188, 240] as follows:

$$\hat{a} = 2.99/(\ln T_{([.9737n]+1)} - \ln T_{([.1673n]+1)}) \quad (1)$$

where n is the size of sample , T_k the kth observation in increasing order and $[x]$ the largest integer not exceeding x . Then $(1.042n^{1/2})(a/\hat{a}-1)$ is approximately distributed as the standard normal Cdf if $n \geq 80$. We are interested in $\hat{a}$ and $\Pr\{a < 1\}$.

Sample spaces are divided into about $1+3.322\log n$ intervals with equally many observations if possible.

The methods of moment , (1) and Gr were used to calculate chisquares. The last one is carried out by feeding pairs (x,y) into a program of linear regression with $x=\ln t$ and $y=\ln\ln(1/\exp(-ct^{a}))=ax+\ln c$. No strict preference could be given to any of them, although Gr yielded smallest and moment procedures largest chisquares when $n \geq 100$. The asymptotic efficiency of $\mathring{a}$ in (1) is .66 . The exactness of fits were remarkably satisfactory ; radar failure time Cdf is often claimed to be exponential.

The shape parameter of Gamma Cdf has an analogous behaviour . By [2,pp165] we know that $\ddot{a}=E^{2}/D$ and $D(\ddot{a})=\sigma^{2}(\ddot{a})=2\ddot{a}(\ddot{a}+1)/n$. Suppose $\ddot{a}<1$ and denote $k=(1-\ddot{a})/\sigma(\ddot{a})$. Then

$$\Pr\{ a<1 \}>1-k^{-2} \qquad (2)$$

as $\Pr\{a\geq 1\}=\Pr\{a-\ddot{a}\geq 1-\ddot{a}\}\leq \Pr\{|a-\ddot{a}|\geq 1-\ddot{a}\}=\Pr\{|a-\ddot{a}|\geq k\sigma(\ddot{a})\}\leq k^{-2}$ the last relation being Chebysev´s inequality. If $n>2\ddot{a}(\ddot{a}+1)/(1-\ddot{a})^{2}$ then $k>1$ and (2) is no triviality. Moreover $D(\ddot{a})<4/n$, $\sigma(\ddot{a})\to 0, k\to\infty, \Pr\{a\geq 1\}\to 0$ when $n\to\infty$ so that the test is consistent, although highly ineffective, especially when $n\leq 100$. Unfortunately Lindhart´s formula [1,pp263] cannot be applied in our case.

In a similar fashion a simple, inefficient test of exactness of fit is formulated as follows: Let a_s, a_k be estimates of a obtained from gamma skewness and kurtosis, respectively, and $k=\max_{x=s,k}\{|a_x-\ddot{a}|\}/\sigma(\ddot{a})$. Then

$$\Pr\{ |a-\ddot{a}|\geq k\sigma(\ddot{a}) \}\leq k^{-2}\to 0 \quad \text{when} \quad n\to\infty \ .$$

2. Determining the Decreasing Failure Rate

Data provided by the local importer, Machinery Co, seemed to show that Raytheon marine radars have decreasing failure rates. Thus, for instance, sample characteristics of Raytheon 1660 TM are by (1): n= 104, å= .727 and Pr{a<1} =N(4)= .9999. At least five different explanations can be offered:

a) the compilation of data from several units with (slightly) different exponential hazard functions may yield a strictly decreasing rate as in [3] ;

b) possible existence of burn-in effects the average age of radars being 3 years;

c) statistical insignificace with n ≤104;

d) radars are better than new after repair or preventive maintenance;

e) renewable components or spares face a continous growth of reliability.

We have been able to reject the first three arguments with the kind aid of the Electrotech. Dep. of the H.Q. Extensive statistics of single radars with more than 100000 h of operational use we collected.

Our data is from four different radars: A I, II, BI and II. By (1) å = .712, .745, .775 and .741 with n= 139,216,424 and 434. In each case Pr{a< 1}≫ .9999. The corresponding values for (2) are a_2=.592.,809,.696, .674 and Pr{a< 1}>.92,.63,.94 and .95.

These results do not favour any preventive maintenance. Radar officers seem to think that serious aging results without it. It must be admitted that no trend of aging can be found in these radars but this is likely to be true also with radars that receive no scheduled over-hauling. The subject is by no means closed, cf pp3 d).

3. Mixtures of Weibull Populations

Radar renewal times are distributed in a complicated way. Several factors such as travel-time and the time needed to obtain spares are not logically related to electronics. Active repair-time can sometimes be fitted by a two or three parameter Weibull Cdf. Mixtures of two or even three 2-parameter Weibull Cdf are required for general downtimes. By writing down the endpoint and number of observations of each successive non-overlapping time-interval we have

AI: .033h,14; .117,19; .183,18; .233,14; .37,18; .8,18; 2.43,19; 7.43,19.

AII: .033h,19; .067,23; .117,32; .25,29, .43,25; 1.83,20; 2.33, 25; 103.25,24.

BI: .05h,42; .1,43; .2,42; .8,43; 1.667,42; 2,43; 3.45,42; 6,43; 7.383,42; 22.033,42.

BII: .033,43; .067,44; .1,43; .133,44; .233,43; .4,43; 1.25, 44; 2,43; 4,43; 200,44.

By denoting Q_k as the weight of the k'th Cdf the following fits were obtained by minimizing chisquares emirically: AI: $a_{1,2}$=6,2.026, $c_{1,2}$=1,24.255, Q_1=.7. AII: $a_{1,2}$= 1.2,5, $c_{1,2}$=9, .015, Q_1=.67. BII: $a_{1,2}$=1.25,1, $c_{1,2}$=13,.4 , Q_1=.55. BI: $a_{1,2,3}$=1.2,6,4 , $c_{1,2,3}$=10,.019,.00038, $Q_{1,2}$=.4,.28.

The corresponding chisquares are 7,7.8,18,3.2 with 2,2,4 and 1 degrees of freedom. Not much has been written about Weibull mixtures, cf [1, pp138-42]. Radar renewal times have been studied by the RADC.

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We inform that the upp- and downtime statistics given by us to Mr Ben Livson, L Phil, are real, measured from practical systems.

Chief of Electrotechnical Department (V Levä)
Colonel V Levä

Bureauengineer (N Forsström)
Major-engineer N Forsström

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